

Introduction to Computer-Based Holography

October 29, 2013 | Carsten Karbach, Jülich Supercomputing Centre (JSC)

Why computergenerated holography?



Source: www.starwars-union.de

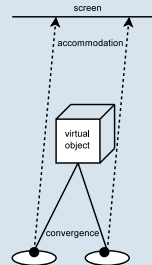
Applications

- **3D Scientific Visualization**
 - **Natural** 3D viewing, focus on different depths
 - **Correct perspective** for multiple viewers
 - No special glasses required
- Holographic optical elements

Problems with Stereoscopy

Accommodation/Convergence conflict

- **Accommodation:**
change power of lens
according to object distance
 - near object → thick lens
 - far object → thin lens
- **Convergence:**
eye rotation to the object



Same perspective for all

- Optimized view for single viewer location
- All viewers receive identical perspective

Challenges for CGH

Software

- Efficient, parallelized implementation
- $O(N^2 * \log(N))$ FLOP per image in real time, with $N = \text{px count of one edge}$, example $N = 10000 \Rightarrow 10^8$
- Multiple 2D Fourier Transformations for each image

Hardware

- Display with extremely **high resolution** required (100* HDTV resolution)
- Pixel distance in magnitude of light waves

Content

- 1 Holography
- 2 Computergenerated Holography (CGH)
- 3 CGH examples

Part I: Holography

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Holography

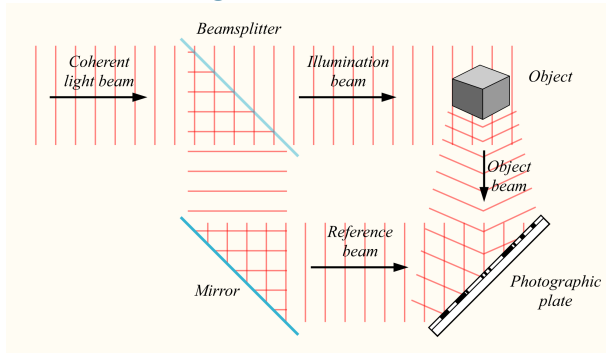
Definition

- Record entire light properties reflected from an object
- Store **intensity** and **phase** information
- Reconstruction of light field of the recorded object

History

- 1948 Dennis Gabor generates first hologram
- 1959 Improved hologram quality by Leith and Upatnieks
- 1960 Invention of laser, requirement for holograms
- 1966 First implemented algorithm for computergenerated hologram

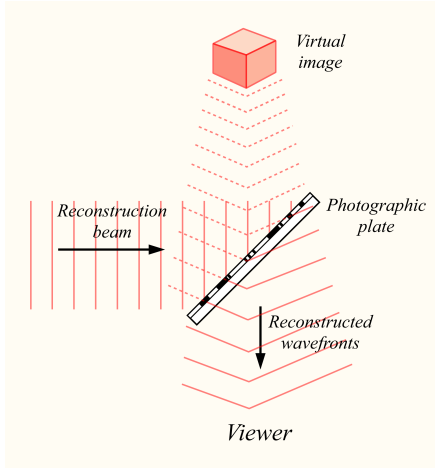
Hologram Recording



Source: <http://en.wikipedia.org/wiki/Holography>

- **Interference** of reference and object beam
- **Coherent** laser light generates standing waves
- Phase and amplitude of light are recorded

Hologram Reconstruction

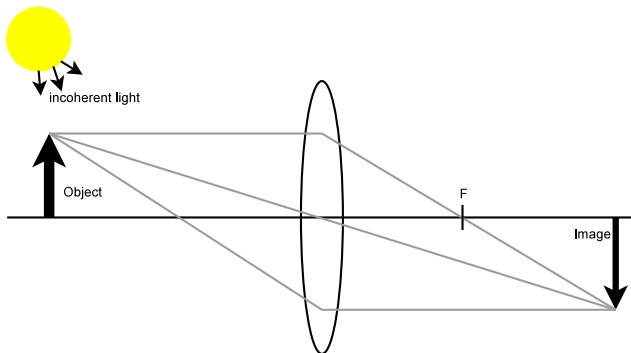


- Reconstruction beam is identical to reference beam
- **Diffraction** of reference beam by hologram
- **Virtual image** appears at position of recorded object
- **Real image** appears in front of the hologram

Source: [http:](http://en.wikipedia.org/wiki/Holography)

[//en.wikipedia.org/wiki/Holography](http://en.wikipedia.org/wiki/Holography)

Comparison Photography

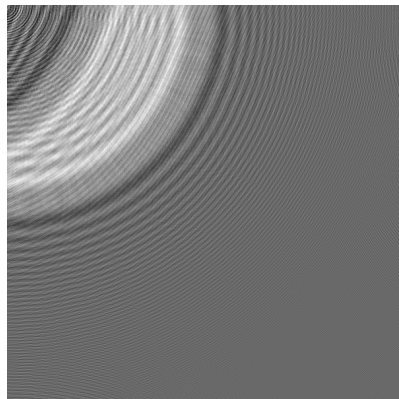


- Incoherent light source
- Lens required for focusing the object

Example hologram

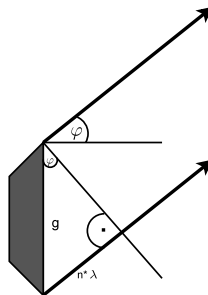
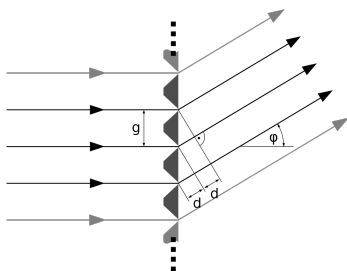


Recorded object



Calculated fringe pattern

Diffraction grating



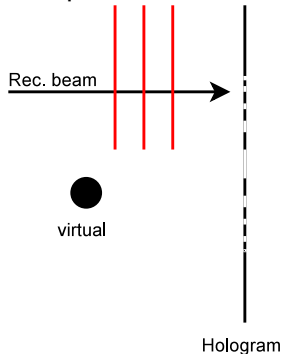
Source: http://de.wikipedia.org/wiki/Optisches_Gitter

- Use of **Huygens-Fresnel** principle leads to generation of plane wave
- Direction of plane wave calculated by $|\sin(\varphi_n)| = \frac{n\lambda}{g}$
 \Rightarrow diffraction grating generates plane waves in directions

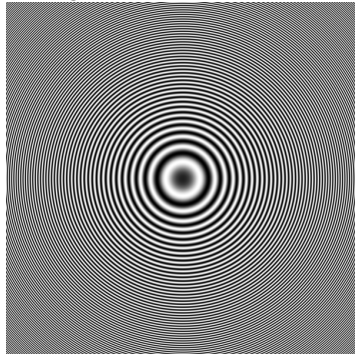
$$\varphi_1, \varphi_{-1}, \varphi_2, \varphi_{-2}, \dots$$

Hologram of one point

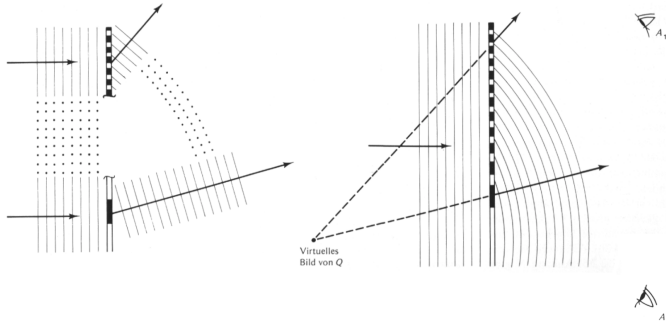
Setup



Hologram



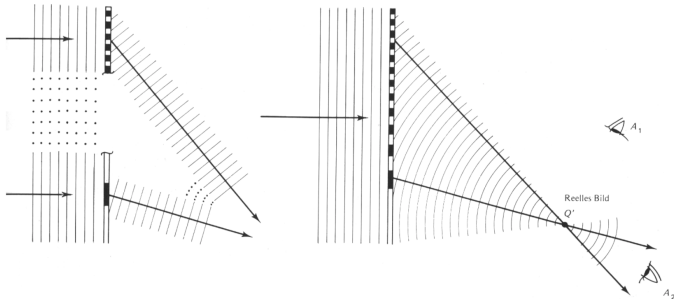
Reconstruction of virtual point



Source: <http://www.holografie.com/Fouquier.pdf>

- Assumption: Hologram is **superposition** of many diffraction gratings
- Each diffraction grating generates plane wave in given direction (here defined by φ_1)

Reconstruction of real point



Source: <http://www.holografie.com/Fouquier.pdf>

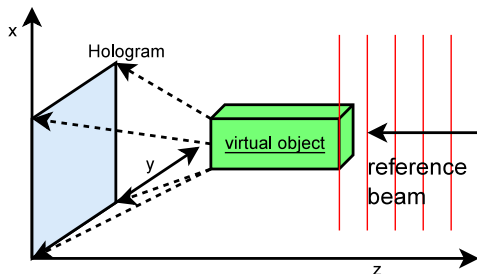
- Diffraction gratings also produce plane waves in φ_{-1} direction
- These plane waves form the real image

Part II: Computergenerated holograms

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Computergenerated holograms

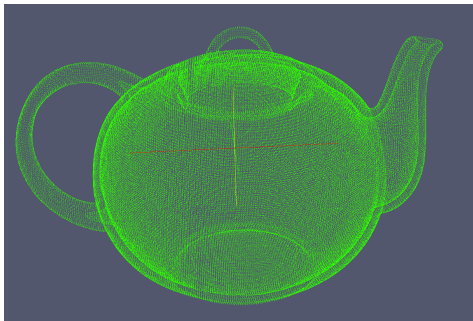
- Calculate fringe pattern on hologram plate
- Create holograms of virtual objects



Instruction

- 1 Describe virtual object
- 2 Calculate light propagation
- 3 Produce hardware hologram
- 4 Hologram reconstruction

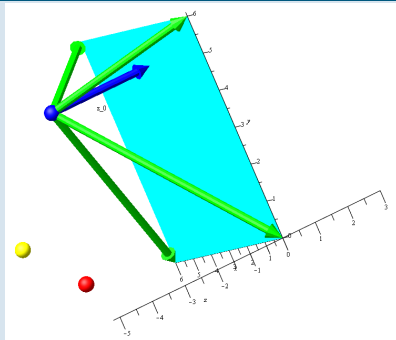
Modeling of virtual scene



- Discretize virtual scene with points
- Use each point as source of a **sphere wave**
- Electric field of sphere wave is $A * \frac{e^{i(\frac{2\pi r}{\lambda} + \phi_0)}}{r}$ with
 r = distance from source to hologram pixel

Calculate CGH

Setting



Calculation

P_l := source of sphere wave l

p_{lm} := pixel(l, m) on hologram plane

$$U_l(P) = \frac{e^{i\frac{2\pi r}{\lambda}}}{r}, \quad r = |P - P_l|$$

$R(P)$ = reference beam = $e^{i\frac{2\pi \langle \vec{k}, P - R_0 \rangle}{\lambda}}$

$H(p_{lm})$ = electric field on hologram plane

$$= R(p_{lm}) + \sum_{i=1}^n U_l(p_{lm})$$

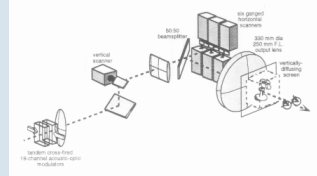
Reconstruction of CGH

Transparency film

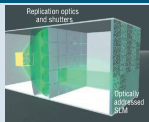


Print CGH with
laser printer

Holovideo system



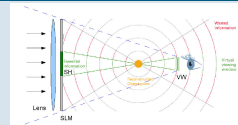
Active Tiling



Modular SLM* with 10^8 pixels

* Spatial light modulator

SeeReal

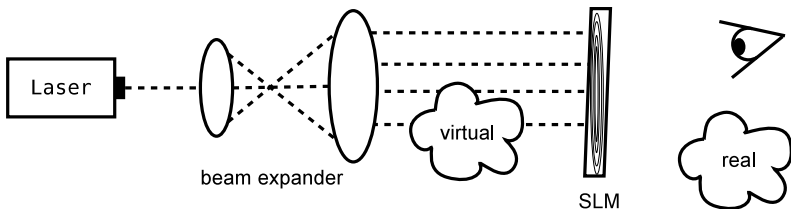


Subhologram

Image sources

- Holovideo: <http://xenia.media.mit.edu/~lucente/holo/36MB/persp.gif>
- Active tiling: Computer-Generated Holography as a Generic Display Technology, C. Slinger et al.
- SeeReal: Large holographic displays as an alternative to stereoscopic displays, R. Häussler et al.

Reconstruction setup



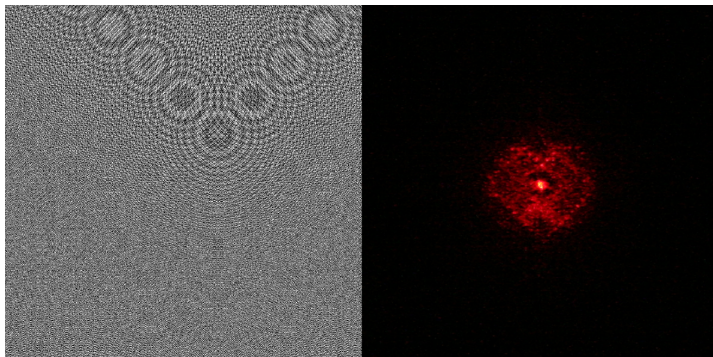
- Laser might be replaced with **LED**
- Light source is required to be **coherent**
- Virtual image visible behind SLM, real image in front of SLM

Part III: CGH examples

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Transparency film

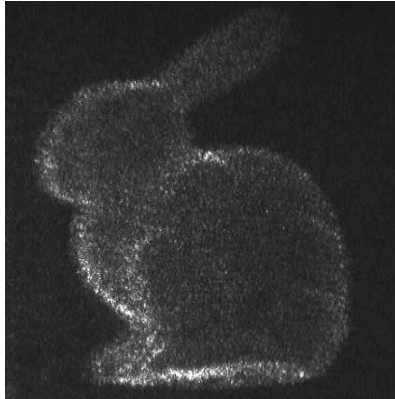
- CGH printed with **laser printer**, reconstructed with **laser pointer**



Source: Computer Generated Holography, J. Wendt

CGH with GPUs

- CGH calculated with **graphic hardware**, reconstruction with Brillian 1080 SLM

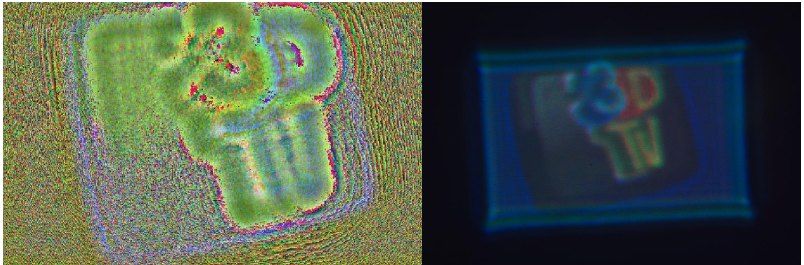


Source: Computer generated holography using parallel commodity graphics hardware, L. Ahrenberg et al.

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Full color CGH

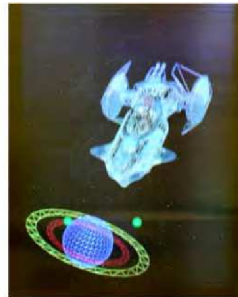
- Three layer CGH, reconstructed with **LEDs** and HoloEye SLM



Source: Color Holographic Reconstruction Using Multiple SLMs and LED Illumination, F. Yaras et al.

SeeReal prototype

- CGH using sub holograms and eye tracking



Source: Holographic 3-D Displays – Electro-holography within the Grasp of Commercialization, S. Reichelt et al.

Summary

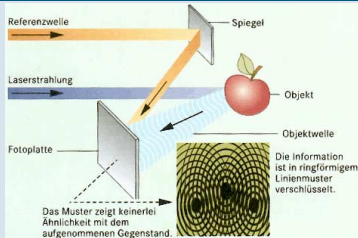
- Holography uses light **interference** and **diffraction** for reconstruction of a 3D-scene
- Promising visualization method
- Modeling concept of holography with sphere waves
- CGH calculates **interference pattern** of virtual scene

Computer-Based Holography – Calculation

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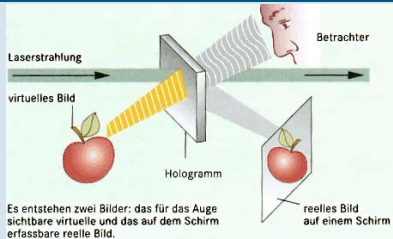
Holography

Recording



- **Interference** of reference and object beam
- Coherent light causes standing waves
- **Phase** and amplitude of light wave are stored

Reconstruction



- **Diffraction** of reconstruction beam on hologram generates object beam
- Reconstruction beam preferably identical to reference beam

Source: <http://www.hobby-output.info/inhalt/hologramm/erklaerung.htm>

Content

- 1 Mathematical derivation
- 2 Light propagation
- 3 Software design

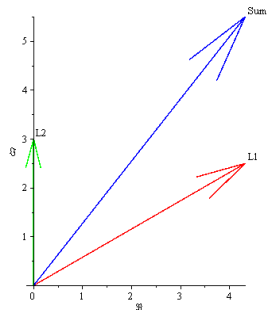
Part I: Simple mathematical derivation

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Modeling of light waves

Complex light

- Light wave defined by **electric field** $E(t)$
- $E(t) = A \cos(\omega t + \phi)$
- Light has amplitude A and phase ϕ
- Complex notation $E(t) = A \exp(i\phi)$ simplifies calculations
- Interference: $A_1 e^{i\phi_1} + A_2 e^{i\phi_2}$
- Intensity: $I = |E(t)|^2 = E(t) * \overline{E}(t)$



Hologram – Derivation I

- Interference of object wave and plane wave of reference beam
- $|H|^2$ is **intensity** stored in hologram

$$\text{Reference beam : } R = A * e^{i\phi}$$

$$\text{Object beam : } O = B * e^{i\sigma}$$

$$\text{Interference : } H = R + O$$

$$\begin{aligned}\Rightarrow |H|^2 &= (R + O) * (\bar{R} + \bar{O}) \\ &= |R|^2 + |O|^2 + O * \bar{R} + R * \bar{O}\end{aligned}$$

Hologram –Derivation II

- Assumption: coherent light

$$\begin{aligned}R * |H|^2 &= R * (|R|^2 + |O|^2 + O * \bar{R} + R * \bar{O}) \\&= R * (|R|^2 + |O|^2) + |R|^2 * O + R^2 * \bar{O} \\&= \text{reference beam} + A^2 * O + R^2 * \bar{O} \\&= \text{reference beam} + \text{virtual image} + \text{real image}\end{aligned}$$

- Diffraction of R by interference pattern $|H|^2$
- Virtual image looks identical to recorded object
- Real image is distracting overlap

Influence of coherent light

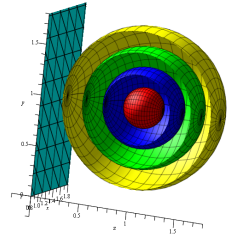
- For incoherent light the phases ϕ and σ vary statistically
- For coherent light the phase difference of both light sources $\phi - \sigma$ is **constant**

$$\begin{aligned}
 O * \bar{R} + R * \bar{O} &= AB(e^{i(\sigma-\phi)} + e^{-i(\sigma-\phi)}) \\
 &= AB(\cos(\sigma - \phi) + i \sin(\sigma - \phi) + \cos(\sigma - \phi) - i \sin(\sigma - \phi)) \\
 &= AB(2 \cos(\sigma - \phi)) \\
 &= \begin{cases} 0, & \text{for incoherent light} \\ AB(2 \cos(\sigma - \phi)), & \text{for coherent light} \end{cases}
 \end{aligned}$$

- With incoherent light the intensity formula is $|H|^2 = |R|^2 + |O|^2$
- Only the intensities of R and O add up, no interference occurs

Sphere wave

- Light wave emitted from a point source P_0
- Uniform propagation of light in **all directions**
- Used for scene points



Formula

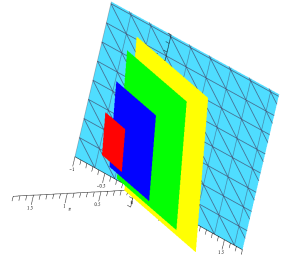
$$U_k(P) = A * \frac{e^{i\left(\frac{2\pi r}{\lambda} + \phi_0\right)}}{r}$$

$$r = |P - P_0|$$



Plane wave

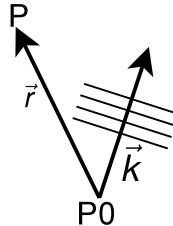
- Light propagates with direction \vec{k}
- Identical phase in each plane perpendicular to \vec{k}
- Used for reference beam



Formula

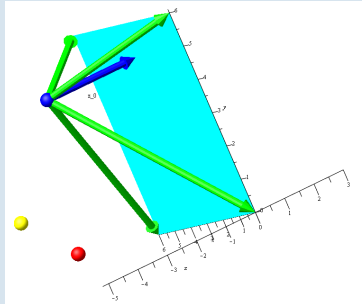
$$U_e(P) = A * e^{i\left(\frac{2\pi\langle\vec{k},\vec{r}\rangle}{\lambda} + \phi_0\right)}$$

$$\vec{r} = \vec{P} - \vec{P}_0$$



Calculate CGHs

Setting



Calculation

P_l := source of sphere wave l

p_{lm} := pixel(l, m) on hologram plane

$$U_l(P) = h_l \frac{e^{j \frac{2\pi r}{\lambda}}}{r}, \quad r = |P - P_l|, \quad h_l = \text{amp}$$

$$R(P) = \text{reference beam} = e^{j \frac{2\pi \langle \vec{k}, \vec{P} - \vec{P}_0 \rangle}{\lambda}}$$

$H(p_{lm})$ = electric field on hologram plane

$$= R(p_{lm}) + \sum_{l=1}^n U_l(p_{lm})$$

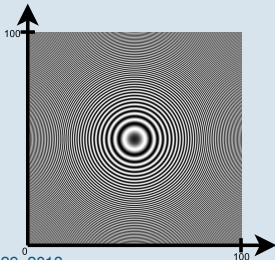
\Rightarrow Calculate $|H(p_{lm})|^2$

for each pixel on hologram plane

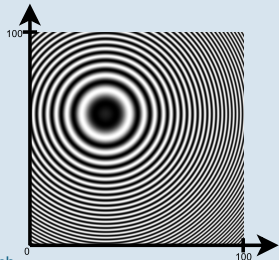
Encoding a 3D-vector in a hologram plane

- Each P_l emits a sphere wave
- Hologram of a single sphere wave generates
Fresnel zone plate
- x and y define the center of the zone plate
- z is encoded in the ring distances

$$P_l = (50, 50, 3)$$



$$P_l = (20, 70, 10)$$



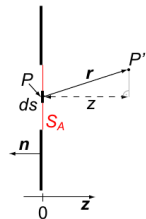
Part II: Light propagation

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Light propagation

Problem

- Calculate light contribution of point P to P'
- Fresnel-Kirchhoff: $U(P') = \frac{1}{i\lambda} \iint_{S_A} U(P) \frac{e^{ikr}}{r} \frac{1}{2} (1 + \frac{\langle \vec{r}, -\vec{n} \rangle}{r}) ds$,
with \vec{n} = normal to plane S_A , $k = \frac{2\pi}{\lambda}$
- Direct calculation is expensive (2D integral for each point P')



Solutions

- 1 **Fresnel** approximation for near field
- 2 **Fraunhofer** approximation for far field
- 3 Use of **Fourier** transformation

Source: *Optische Fouriertransformation*, Westfälische Wilhelms-Universität Münster

Comparison to simple derivation

Fresnel-Kirchhoff

$$U(P') = \frac{1}{i\lambda} \iint_{S_A} U(P) \frac{e^{ikr}}{r} \frac{1}{2} \left(1 + \frac{\langle \vec{r}, -\vec{n} \rangle}{r} \right) ds$$

Discretization of S_A into n points \Rightarrow

$$U(P') \approx \frac{1}{i\lambda} \sum_{l=1}^n U(P_l) \frac{e^{ikr}}{r} \frac{1}{2} \left(1 + \frac{\langle \vec{r}, -\vec{n} \rangle}{r} \right)$$

$-\vec{n} \approx \frac{\vec{r}}{r}$, true for $z \gg x^2 + y^2 \Rightarrow$

$$U(P') \approx \frac{1}{i\lambda} \sum_{l=1}^n U(P_l) \frac{e^{ikr}}{r}$$

Start a sphere wave from each P_l on S_A

Sphere waves

$$H(p_{lm}) = R(p_{lm}) + \sum_{l=1}^n U_l(p_{lm}) =$$

$$R(p_{lm}) + \sum_{l=1}^n h_l \frac{e^{ikr}}{r}$$

Start a sphere wave from each scene point P_l

- Both approaches use **Huygens-Fresnel** principle

Fresnel approximation

Source: *Optische Fouriertransformation*, Westfälische Wilhelms-Universität Münster

- $|x|, |y|, |x'|, |y'| \ll z$
- $\vec{r} = P' - P = \begin{pmatrix} x_h \\ y_h \\ z \end{pmatrix} - \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$
- $|\vec{r}| \overset{(*)}{\approx} z + \frac{(x_h - x)^2}{2z} + \frac{(y_h - y)^2}{2z}$
in phase
- $|\vec{r}| \approx z$ in amplitude,
inaccuracy uncritical

$$\Rightarrow U(P') = \frac{1}{i\lambda z} e^{ikz} \iint_{S_A} U(P) e^{\frac{ik}{2z}((x_h - x)^2 + (y_h - y)^2)} dx dy$$

(*) Approximation for $|\vec{r}|$

- $\sqrt{1+x} = 1 + \frac{x}{2} + O(x^2)$,
Taylor series eval. at $x = 0$
- $|\vec{r}| = \sqrt{z^2 + (x_h - x)^2 + (y_h - y)^2} = z \sqrt{1 + \frac{(x_h - x)^2}{z^2} + \frac{(y_h - y)^2}{z^2}}$
- Use $\frac{(x_h - x)^2}{z^2} + \frac{(y_h - y)^2}{z^2}$ as x in Taylor series

$$\Rightarrow |\vec{r}| \approx z \left(1 + \frac{(x_h - x)^2}{2z^2} + \frac{(y_h - y)^2}{2z^2} \right)$$

Fraunhofer approximation

Source: *Optische Fouriertransformation*, Westfälische Wilhelms-Universität Münster

- With $z \gg x^2 + y^2 \Rightarrow |\vec{r}| = z + \frac{x_h^2 + y_h^2}{2z} - \frac{x_h x + y_h y}{z} + \frac{x^2 + y^2}{2z}$
 $\approx z + \frac{x_h^2 + y_h^2}{2z} - \frac{x_h x + y_h y}{z}$, for phase
- $$U(P') = \frac{1}{i\lambda z} e^{ikz} e^{\frac{ik}{2z}(x_h^2 + y_h^2)} \iint_{S_A} U(P) e^{\frac{-ik}{z}(x_h x + y_h y)} dx dy$$
- Advantage:
$$\iint_{S_A} U(P) e^{\frac{-ik}{z}(x_h x + y_h y)} dx dy = FT(U)\left(\frac{x_h}{z\lambda}, \frac{y_h}{z\lambda}\right)$$
- Complexity of calculation is proportional to 2D Fourier transformation

Use of Fourier transformation

Source: *Optische Fouriertransformation*, Westfälische Wilhelms-Universität Münster

- 2D FT: $F(u, v) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} f(x, y) * e^{-i2\pi (x u + y v)} dx dy$

- Goal: use FFT in hologram generation

- Start with Fraunhofer approximation:

$$\int\limits_{S_A} U(P) e^{\frac{-ik}{z}(x_h x + y_h y)} dx dy =$$

$$\int\limits_{S_A} U(P(x, y)) e^{-i2\pi(\frac{x_h}{z\lambda} x + \frac{y_h}{z\lambda} y)} dx dy = F(U)(\frac{x_h}{z\lambda}, \frac{y_h}{z\lambda})$$

- With more effort FT can also be used for Fresnel approximation

FT for Fresnel

Source: *Optische Fouriertransformation*, Westfälische Wilhelms-Universität Münster

$$\begin{aligned}
 \iint_{S_A} U(P) e^{\frac{ik}{2z} ((x_h-x)^2 + (y_h-y)^2)} dx dy &= \\
 \iint_{S_A} U(P) e^{\frac{i2\pi}{\lambda 2z} (x_h^2 - 2xx_h + x^2 + y_h^2 \dots)} dx dy &= \\
 e^{i \frac{\pi}{\lambda z} (x_h^2 + y_h^2)} \iint_{S_A} U(P) e^{\frac{i\pi}{\lambda z} (x^2 + y^2)} e^{-i \frac{2\pi}{z\lambda} (xx_h + yy_h)} dx dy &= \\
 e^{i \frac{\pi}{\lambda z} (x_h^2 + y_h^2)} F\left(U(P) e^{\frac{i\pi}{\lambda z} (x^2 + y^2)} \right) \left(\frac{x_h}{z\lambda}, \frac{y_h}{z\lambda} \right) &=
 \end{aligned}$$

- Expanding the formula, and extracting the parts depending on x and y leads to a formula using Fourier transformation

Numerical reconstruction

- **Inverse operation** to hologram generation
- Calculate original light wave from a hologram
- Input is either a CGH or
a digital hologram captured with a CCD
- Allows to calculate amplitude
and phase of source light wave
- Calculation of light wave front at arbitrary location

Numerical reconstruction – Idea

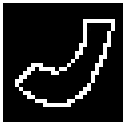
- Almost identical to normal hologram calculation!
- **Diffraction** of reference beam on hologram
- Use Fresnel-Kirchhoff or Fresnel-approximation:

$$U(x, y, z) = \frac{1}{i\lambda z} e^{i\frac{2\pi z}{\lambda}} \iint_{S_A} h(x_h, y_h) * e^{i\frac{\pi}{\lambda z}((x-x_h)^2+(y-y_h)^2)} dx_h dy_h$$

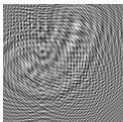
- Reconstruction of original wave in any plane parallel to hologram plane
- Discretization of integral leads to: $U(x, y, z) = \frac{1}{i\lambda z} e^{i\frac{2\pi z}{\lambda}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} h(x_k, y_l) * e^{i\frac{\pi}{\lambda z}((x-x_k)^2+(y-y_l)^2)}$

Numerical reconstruction – Example

- 1 Choose original object



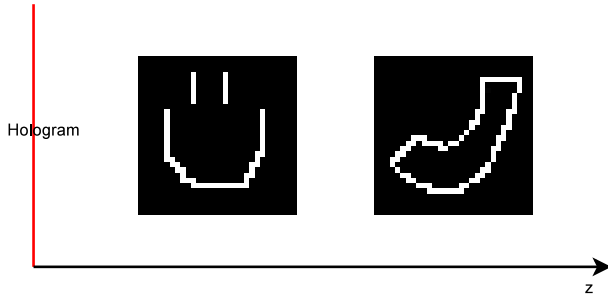
- 2 Calculate CGH e.g. via Fresnel-approximation



- 3 Use CGH amplitude $h(x_h, y_h)$ and reconstruct



Numerical reconstruction 3D example



Reconstruction:



Part III: Software design

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Algorithm

```
Complex* generateHologram(points, dim)

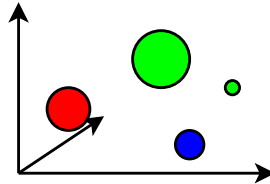
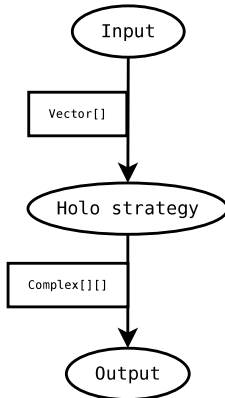
result = new Complex[dim*dim]

for(y=0; y < dim; y++)
    for(x=0; x < dim; x++)
        platePoint(x/pixelPerMeterX, y/pixelPerMeterY, 0.0)
        for(i=0; i<points.size(); i++)
            point = points[i]
            distance = platePoint.getDistance(point), phase = 2*PI/lambda*distance
            amplitude = point.getAmplitude()/distance
            contribution.setReal( amplitude*cos(phase) )
            contribution.setImag( amplitude*sin(phase) )
            result[y*dim+x] += contribution

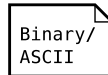
return result
```

- Use of simple derivation, sphere waves
- Input:
 - **points**: source points
 - **dim**: pixels for width and height
- Output is array of Complex values
- Fresnel/Fraunhofer approximation can also be used here

CGH application



Raycast / Fresnel / Fraunhofer
Fourier



CGH application – remarks

- Comparable with **VTK visualization pipeline**
- Input is a list of 3D vectors for **point sources**
 - generated
 - from file
 - 2D image as source
- **Strategies** for CGH calculation:
 - Raycast: Sum of sphere waves
 - Fresnel: Raycast using Fresnel approximation, avoids square roots
 - Fraunhofer: Raycast with Fraunhofer approximation
 - *Fourier*: Discretize volume into planes, run 2D DFT for each plane
- **Output** 2D image, binary / XML file

Parallelization

Approaches

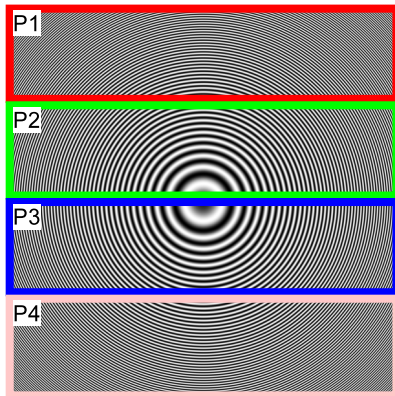
- 1 **Domain** decomposition of 2D hologram
- 2 Decomposition of **scene points**, global sum
- 3 **Time** decomposition for dynamic images

Implementation

- Java prototype for testing
- C++ with MPI/CUDA implementation of approach 1 as efficient solution

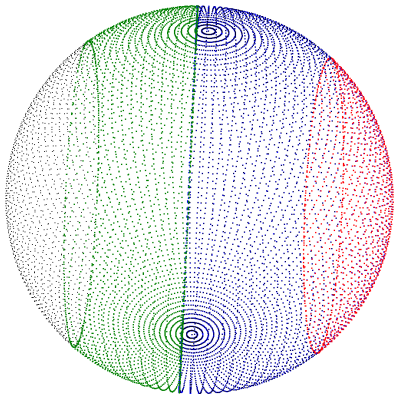
Domain decomposition

- Distribute image space uniformly on all CPUs
- Embarrassingly parallel, no communication required
- Target: interactive frame rates, short response times



Decomposition of scene points

- Distribute scene points across the CPUs
- **Global sum** of all contribution parts afterwards



OpenGL adapter

- 1 Get transformed vertices from the underlying visualization application
- 2 Optionally add more vertices by **interpolation**
- 3 Use these vertices as source points in hologram calculation
- 4 Render **interference pattern** onto screen
- 5 Use the screen as spatial light modulator

Remarks

- Simple adaption of existing visualization tools to CGH
- New input type for CGH application

Part IV: Outlook

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- ```
Complex* generateHistogram(points, dim)

result = new Complex(dim*dim);

for(int i; i < dim; i++)

{
 for(int j; j < dim; j++)

 {
 plotPoint.x(plotMeanX, plotMeanY, 0.0);

 for(int k; k < points.size(); k++)

 {
 point = points[k];

 distance = plotPoint.getDistance(point);
 phase = 2*M_PI*angle(x);

 amplitude = point.getAmplitude();

 contribution = (k * amplitude * cos(phase));
 }
 }
}
```

## Outlook

- Performance analysis of parallel implementations for CGH generation
- Requirement for reconstruction: **affordable SLM**
- Evaluation of hologram calculation using **FFT**
- Integration of CGH rendering into existing visualization software (OpenGL adapter)